Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Thursday 4 May 2017 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

24EP01

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Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1.   Consider the numbers $p = 2.78 \times 10^{11}$ and $q = 3.12 \times 10^{-3}$.

   (a)   Calculate $\sqrt[3]{\dfrac{p}{q}}$. Give your full calculator display.   [2]

   (b)   Write down your answer to part (a)

       (i)   correct to two decimal places;

       (ii)   correct to three significant figures.   [2]

   (c)   Write your answer to **part (b)(ii)** in the form $a \times 10^{k}$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.   [2]

Working:

Answers:

(a)   . . . .

(b)   (i)   . . . .

       (ii)   . . . .

(c)   . . . .

2. All the children in a summer camp play at least one sport, from a choice of football (F) or basketball (B). 15 children play both sports.

The number of children who play only football is double the number of children who play only basketball.

Let x be the number of children who play only football.

(a) Write down an expression, in terms of x, for the number of children who play only basketball. [1]

(b) Complete the Venn diagram using the above information. [2]
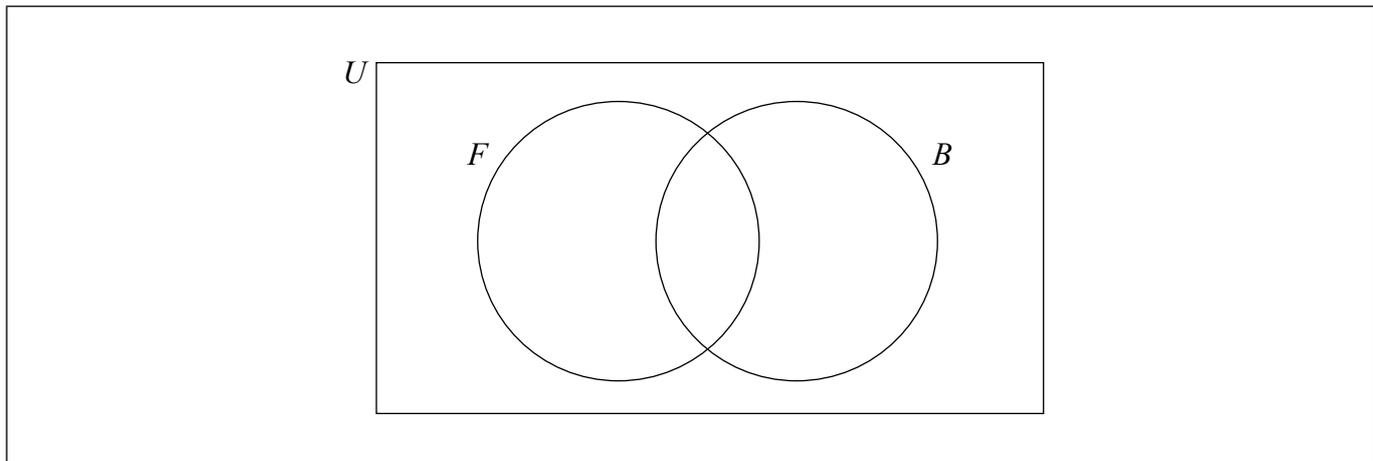
There are 120 children in the summer camp.

(c) Find the number of children who play only football. [2]

(d) Write down the value of $n(F)$. [1]

Working:

Answers:

(a) ....

(c) ....

(d) ....

Turn over

3. A lampshade, in the shape of a cone, has a wireframe consisting of a circular ring and four straight pieces of equal length, attached to the ring at points A, B, C and D.

The ring has its centre at point O and its radius is 20 centimetres. The straight pieces meet at point V, which is vertically above O, and the angle they make with the base of the lampshade is 60°.
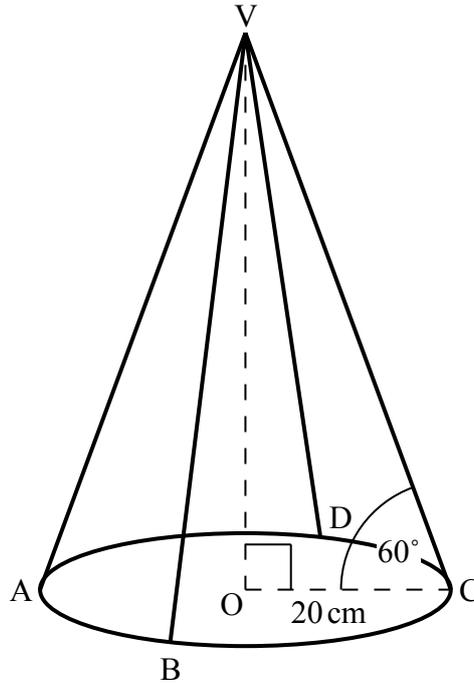
This information is shown in the following diagram.

diagram not to scale

(a) Find the length of one of the straight pieces in the wireframe. [2]

(b) Find the total length of wire needed to construct this wireframe. Give your answer in centimetres correct to the nearest millimetre. [4]

(This question continues on the following page)

24EP04

(Question 3 continued)

Working:

Answers:

(a) . . . .

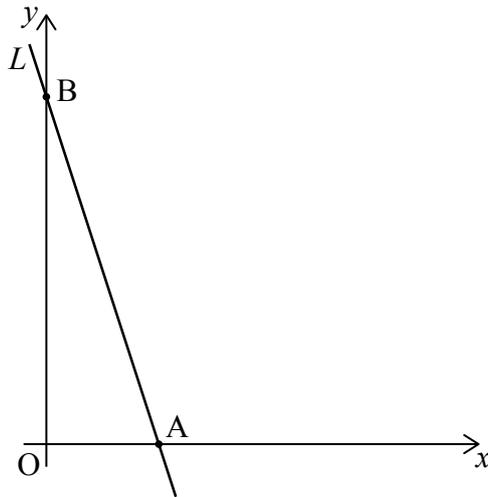(b) . . . .

4. Line L intersects the x-axis at point A and the y-axis at point B, as shown on the diagram.

The length of line segment OB is three times the length of line segment OA, where O is the origin.

(a) Find the gradient of L. [2]

Point $(2, 6)$ lies on L.

(b) Find the equation of L in the form $y = mx + c$. [2]

(c) Find the x-coordinate of point A. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

5. Tomás is playing with sticks and he forms the first three diagrams of a pattern. These diagrams are shown below.
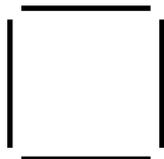
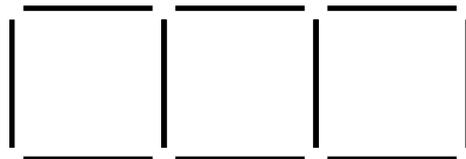Diagram 1   Diagram 2   Diagram 3

Tomás continues forming diagrams following this pattern.

(a) Diagram n is formed with 52 sticks. Find the value of n. [3]

Tomás forms a total of 24 diagrams.

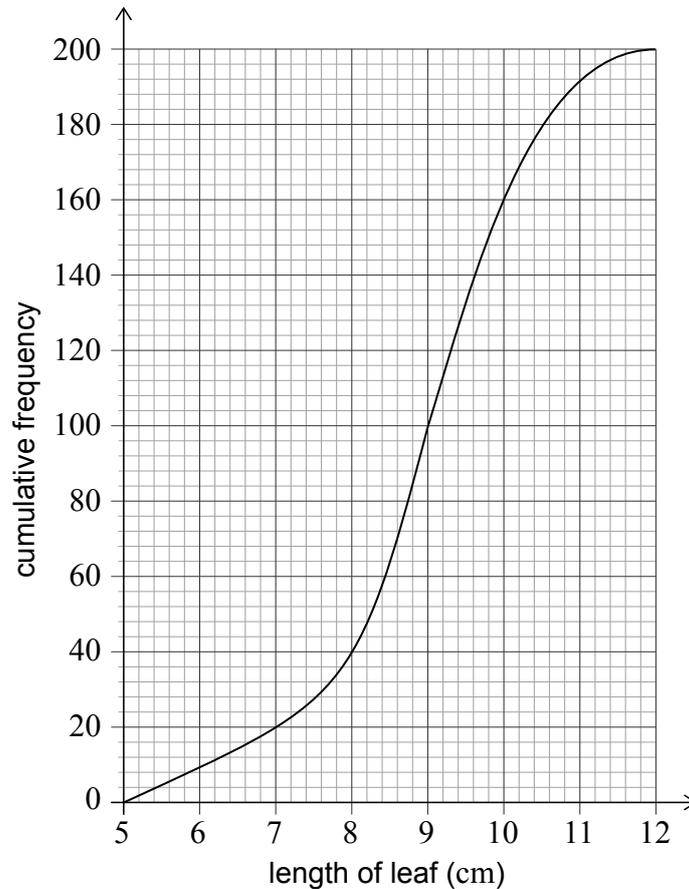(b) Find the total number of sticks used by Tomás for all 24 diagrams. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

Turn over

6. For a study, a researcher collected 200 leaves from oak trees. After measuring the lengths of the leaves, in cm, she produced the following cumulative frequency graph.

(a)   Write down the median length of these leaves.   [1]

(b)   Write down the number of leaves with a length less than or equal to 8 cm.   [1]

The researcher finds that 10% of the leaves have a length greater than k cm.

(c)   (i)   Use the graph to find the value of k.

(ii)   Before measuring, the researcher estimated k to be approximately 9.5 cm. Find the percentage error in her estimate.   [4]

(This question continues on the following page)

(Question 6 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) (i) . . . .

(ii) . . . .

7.   A tetrahedral (four-sided) die has written on it the numbers 1, 2, 3 and 4. The die is rolled many times and the scores are noted. The table below shows the resulting frequency distribution.

Score	1	2	3	4
Frequency	18	x	y	22

The die was rolled a total of 100 times.

(a)   Write down an equation, in terms of x and y, for the total number of times the die was rolled.   [1]

The mean score is 2.71.

(b)   Using the mean score, write down a second equation in terms of x and y.   [2]

(c)   Find the value of x and of y.   [3]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

8. Claudia travels from Buenos Aires to Barcelona. She exchanges 8000 Argentine Pesos (ARS) into Euros (EUR).

The exchange rate is $1\,\text{ARS} = 0.09819\,\text{EUR}$. The bank charges a 2% commission on the exchange.

(a) Find the amount of Euros that Claudia receives. Give your answer correct to two decimal places. [3]

When Claudia returns to Buenos Aires she has 85 EUR left and exchanges this money back into ARS. The exchange rate is $1\,\text{ARS} = 0.08753\,\text{EUR}$. The bank charges r% commission. The commission charged on this exchange is 14.57 ARS.

(b) Find the value of r. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

Turn over

9. Consider the geometric sequence $u_1 = 18$, $u_2 = 9$, $u_3 = 4.5$, ….

(a) Write down the common ratio of the sequence. [1]

(b) Find the value of u_5. [2]

(c) Find the smallest value of n for which u_n is less than 10^{-3}. [3]

Working:

Answers:

(a) ....

(b) ....

(c) ....

10. The Home Shine factory produces light bulbs, 7% of which are found to be defective.

(a) Write down the probability that a light bulb produced by Home Shine is not defective. [1]

Francesco buys two light bulbs produced by Home Shine.

(b) (i) Find the probability that both light bulbs are not defective.

(ii) Find the probability that at least one of Francesco's light bulbs is defective. [4]

The Bright Light factory also produces light bulbs. The probability that a light bulb produced by Bright Light is not defective is a.

Deborah buys three light bulbs produced by Bright Light.

(c) Write down an expression, in terms of a, for the probability that at least one of Deborah's three light bulbs is defective. [1]

Working:

Answers:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

(c) . . . .

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Answers written on this page will not
be marked.

11. The mass of a certain type of Chilean corncob follows a normal distribution with a mean of 400 grams and a standard deviation of 50 grams.

(a) Write down the probability that the mass of one of these corncobs is greater than 400 grams. [1]

A farmer labels one of these corncobs as premium if its mass is greater than a grams. 25 % of these corncobs are labelled as premium.

(b) Find the value of a. [2]

(c) Estimate the interquartile range of the distribution. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

12. A cylindrical container with a radius of $8\,\text{cm}$ is placed on a flat surface. The container is filled with water to a height of $12\,\text{cm}$, as shown in the following diagram.

diagram not to scale

$12\,\text{cm}$

$8\,\text{cm}$
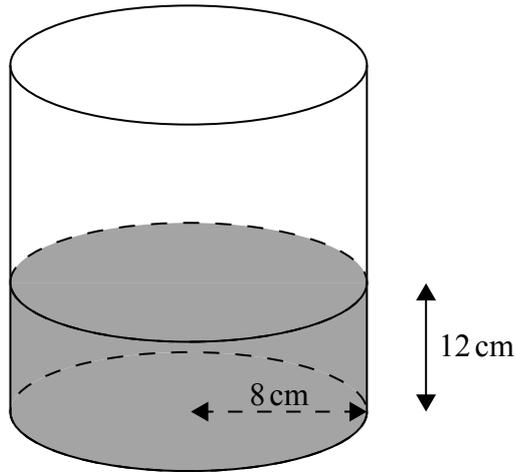
(a) Find the volume of water in the container. [2]

A heavy ball with a radius of $2.9\,\text{cm}$ is dropped into the container. As a result, the height of the water increases to h cm, as shown in the following diagram.
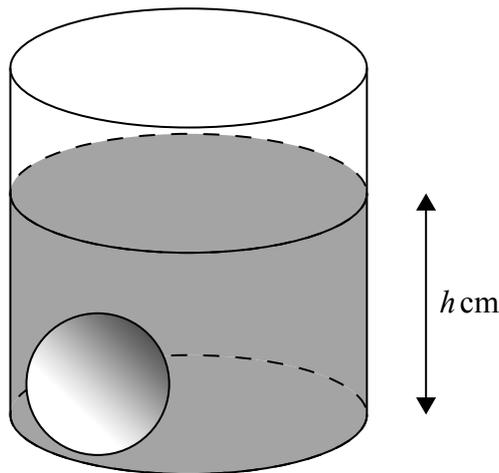
diagram not to scale

$h\,\text{cm}$

(b) Find the value of h. [4]

(This question continues on the following page)

(Question 12 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

Turn over

13. The diagram shows part of the graph of a function $y = f(x)$. The graph passes through point $A(1, 3)$.
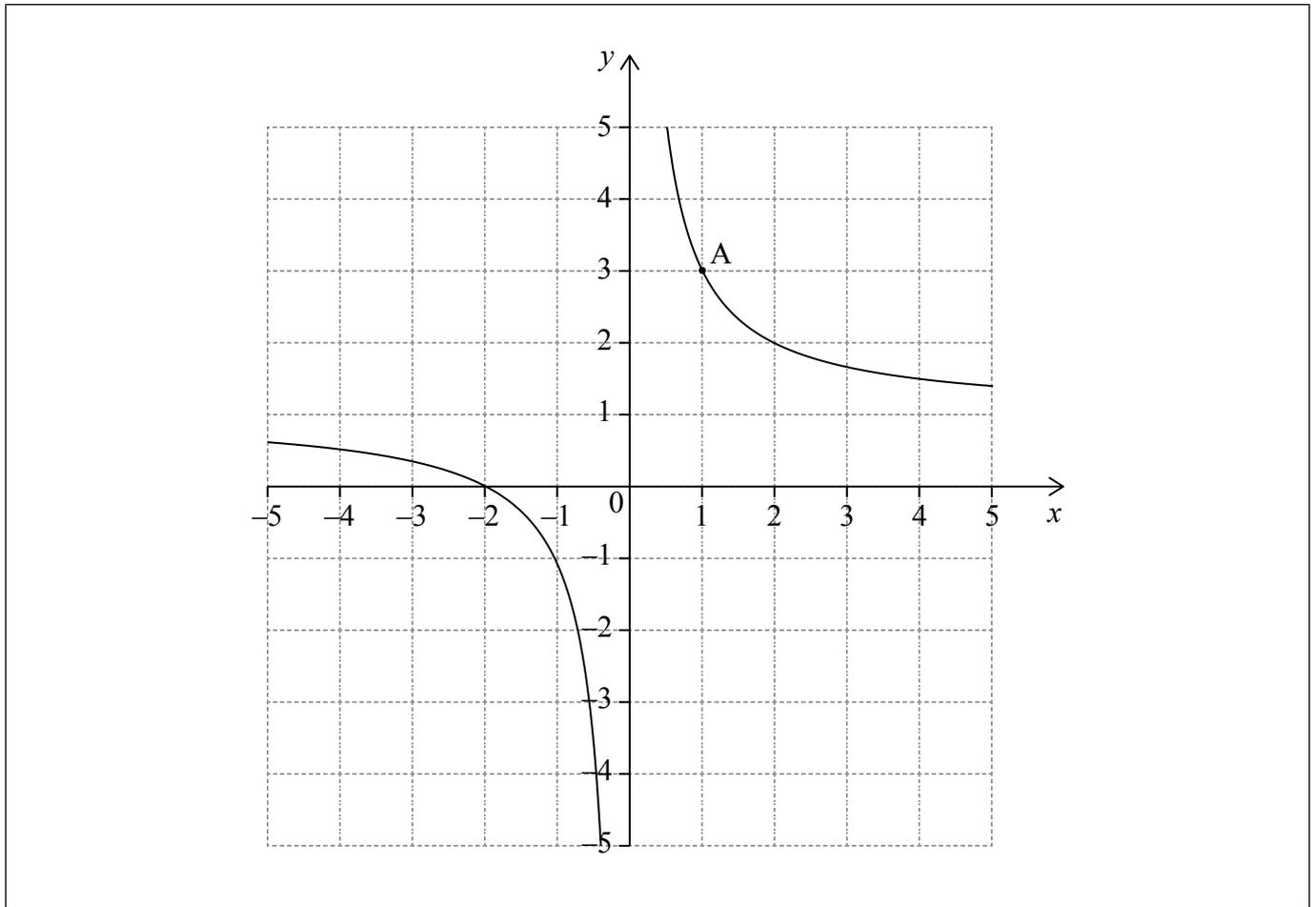
(a) Write down the value of $f(1)$. [1]

The tangent to the graph of $y = f(x)$ at A has equation $y = -2x + 5$. Let N be the normal to the graph of $y = f(x)$ at A.

(b) Find the equation of N. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) Draw the line N on the diagram above. [2]

(This question continues on the following page)

(Question 13 continued)

Working:

Answers:

(a) ....

(b) ....

Turn over

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be marked.

14. Jashanti is saving money to buy a car. The price of the car, in US Dollars (USD), can be modelled by the equation

$$P = 8500\,(0.95)^t.$$

Jashanti's savings, in USD, can be modelled by the equation

$$S = 400t + 2000.$$

In both equations t is the time in months since Jashanti started saving for the car.

(a)   Write down the amount of money Jashanti saves per month.   [1]

(b)   Use your graphic display calculator to find how long it will take for Jashanti to have saved enough money to buy the car.   [2]

Jashanti does not want to wait too long and wants to buy the car two months after she started saving. She decides to ask her parents for the extra money that she needs.

(c)   Calculate how much extra money Jashanti needs.   [3]

Working:

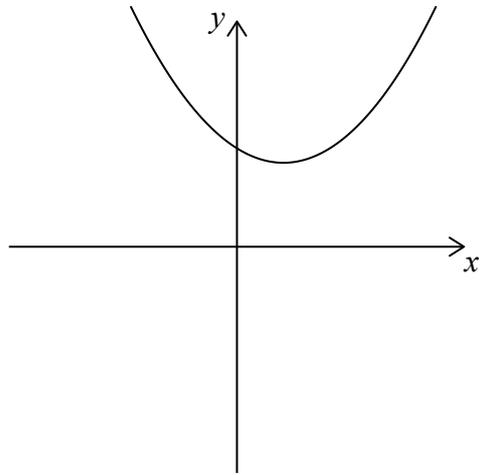**Answers:**

(a) . . . .

(b) . . . .

(c) . . . .

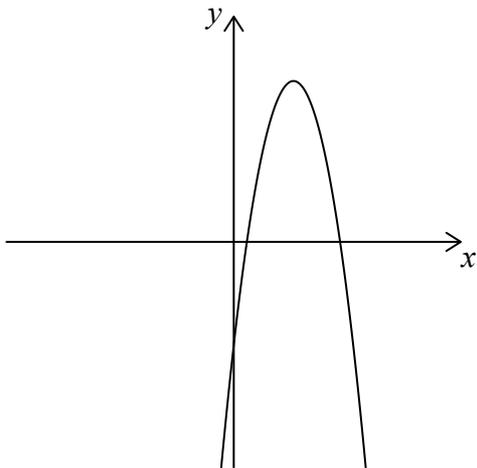**15.** Consider the following graphs of quadratic functions.
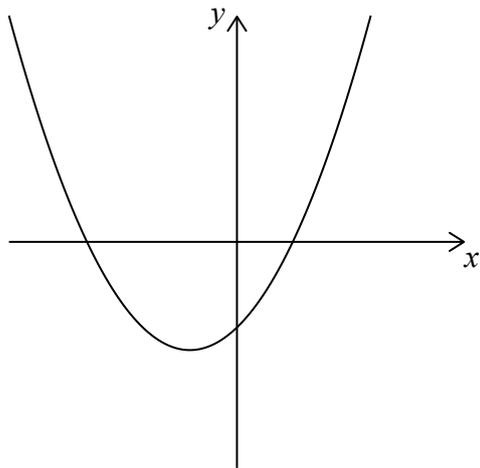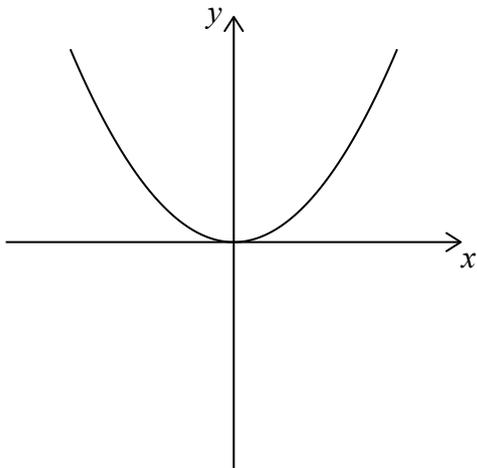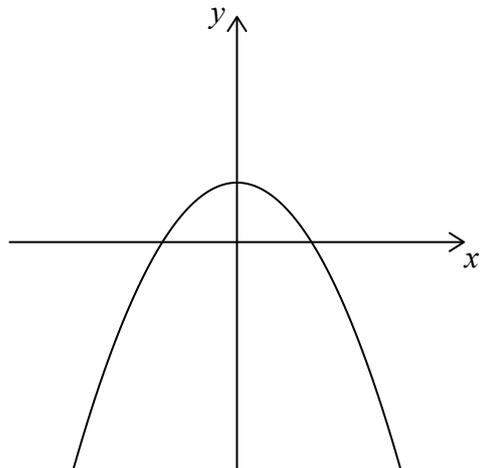
Graph 1.
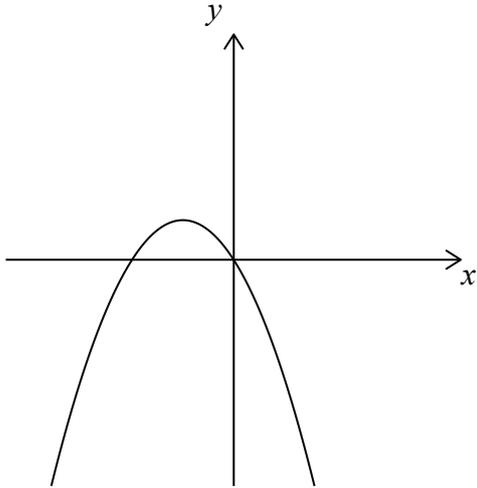
Graph 2.

Graph 3.

Graph 4.

Graph 5.

Graph 6.

(This question continues on the following page)

(Question 15 continued)

The equation of each of the quadratic functions can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

Each of the sets of conditions for the constants a, b and c, in the table below, corresponds to one of the graphs on the opposite page.

Write down the number of the corresponding graph next to each set of conditions. [6]

Conditions	Graph number
$a > 0$, $b < 0$, $c > 0$	
$a < 0$, $b = 0$, $c > 0$	
$a < 0$, $b > 0$, $c < 0$	
$a > 0$, $b = 0$, $c = 0$	
$a > 0$, $b > 0$, $c < 0$	
$a < 0$, $b < 0$, $c = 0$	

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